# Appendix C - Forming a Linear Equation on a Grid Node

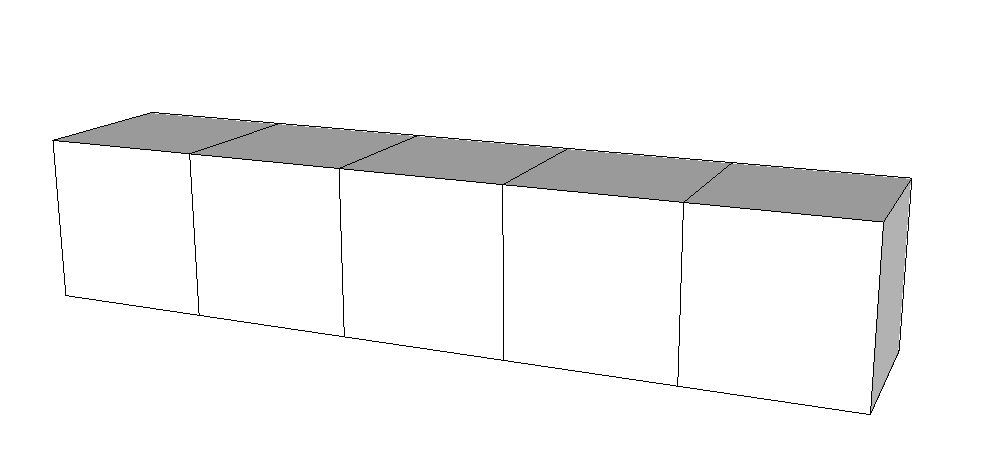
Again we consider **Eq. A4**, with a bit rearrangement,

Each of these differential terms (with respect to , , , and ) is translated accordingly to form the diffusivity equation for one particular point. This will form a linear equation with some unknown terms and known variables. Notice that we will get the term only when performing differential with respect to . Also, we will get a known term when performing differential with respect to . Therefore, we rearrange the equation further as follows,

where:

We can form the following matrices,

# Appendix D - Analytical Solution to 1D- Flow (Example 1)



**• 1**

**• 2**

**• 3**

**• 4**

**• 0**

75 ft

1000 ft

1000 ft

Figure - Porous medium and grid block system for Example 1

For the 1D-flow through gridblocks as shown in **Fig. 3**, determine the pressure distribution during the first year of production. The initial reservoir pressure is . The rock and fluid properties for this problem are , , , , , , , and .

Consider the following partial differential equation (as derived in **App. A** with no sink/source term) with its initial and boundary condition

|  |  |
| --- | --- |
| Partial differential equation (PDE) | |
|  | |
| Initial condition (IC) | Boundary condition (BC) |
|  |  |

First, we need to assume that the only variable that changes with respect to and is . All else stay constant. We can rearrange the PDE as follows.

where .

***Step 1. Method of Separating Variables***

By **method of separating variables**, we are trying to construct a solution of the form , that is . The PDE then becomes

Both sides must be constant. Because say we would like to vary with respect to without varying , the left hand side which contains terms must stay constant. Conversely, varying only the term can only be done while the right hand side is left unaltered. Say its constant is , we can then get two ordinary differential equations (ODE).

***Step 2. Satisfying Boundary Conditions***

1. **Evaluating**

We are trying to construct a solution of the form .

Therefore, we get

1. **Evaluating**

We are trying to construct a solution of the form .

We have a quadratic equation as its characteristic equation (). The form of is determined by the separation constant . If , then and the solution will be . From the boundary condition, one can show that , and we will eventually get a zero solution , which will further lead to . If , say , a general solution will be

Again, from the boundary condition, one will eventually obtain as before. We are now left with , say . We will get complex solutions as follows,

By **Euler’s formula**, we can express in terms of real solutions,

After carefully evaluating and for this problem, we arrive at the following general solution,